Thermal Models in High Energy Physics.

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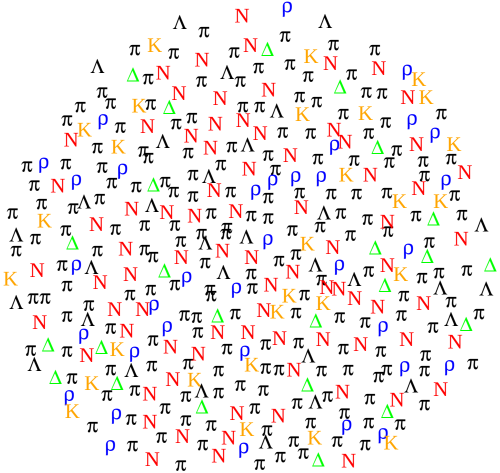
6th EU-Russia-JINR@Dubna Round Table Discussion,
3 - 5 March 2014

Topics:

- **Thermal Models from 1 to 7000 GeV**
- $\sqrt{s_{NN}}$ **= 11 GeV is special**
- **The Tsallis Distribution at the LHC**

Hadronic Gas



J.C. and H. Satz, Z. fuer Physik C57, 135, 1993.

	Equilibrium
π	$\exp\left[-\frac{E_\pi}{T}\right]$
N	$\exp\left[-\frac{E_N}{T}+\frac{\mu_B}{T}\right]$
$\overline{N}$	$\exp\left[-\frac{E_N}{T}-\frac{\mu_B}{T}\right]$
Λ	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_B}{T}-\frac{\mu_S}{T}\right]$
$\overline{\Lambda}$	$\exp\left[-\frac{E_\Lambda}{T}-\frac{\mu_B}{T}+\frac{\mu_S}{T}\right]$
K	$\exp\left[-\frac{E_K}{T}+\frac{\mu_S}{T}\right]$
$\overline{K}$	$\exp\left[-\frac{E_K}{T}-\frac{\mu_S}{T}\right]$

SPS data.

	Measurement
Pb–Pb 158*A* GeV	
$(\pi^+ + \pi^-)/2.$	600±30
K^+	95 ±10
K^-	50 ±5
K^0_S	60 ±12
p	140±12
$\bar{p}$	10 ±1.7
ϕ	7.6±1.1
Ξ^-	4.42±0.31
$\overline{\Xi^-}$	0.74±0.04
$\bar{\Lambda}/\Lambda$	0.2±0.04

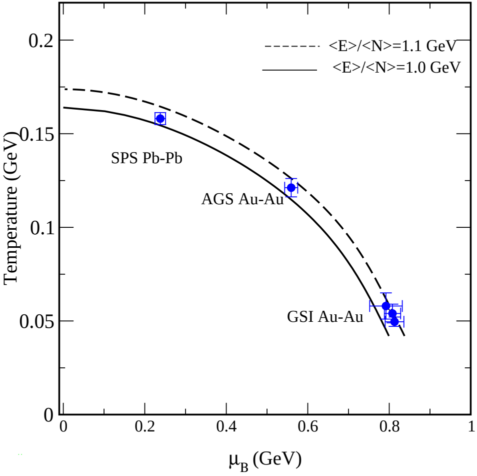
SPS data.

SPS: Freeze-Out Parameters:

$$T = 156.0 \pm 2.4\mathrm{MeV}$$
$$\mu_B = 239 \pm 12\mathrm{MeV}$$

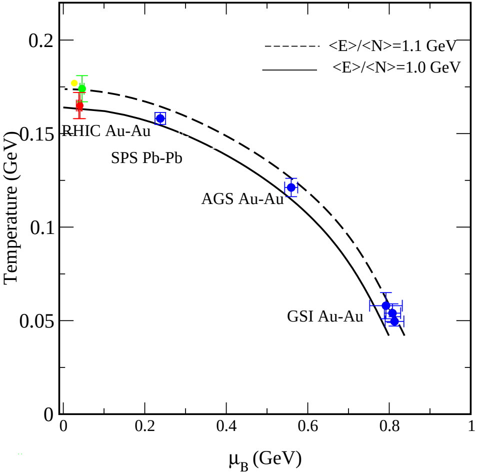
F. Becattini, J.C., A. Keränen, E. Suhonen and K. Redlich
Physical Review C64 (2001) 024901.

E/N in 1999



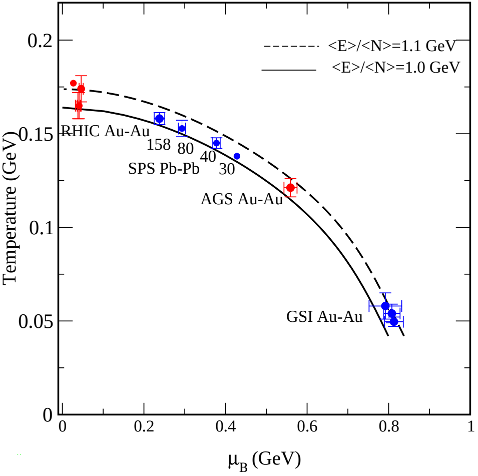
<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
SPS Pb-Pb
AGS Au-Au
GSI Au-Au
Temperature (GeV)
0
0.05
0.1
0.15
0.2
0
0.2
0.4
0.6
0.8
1
μ_B (GeV)

E/N in 2000



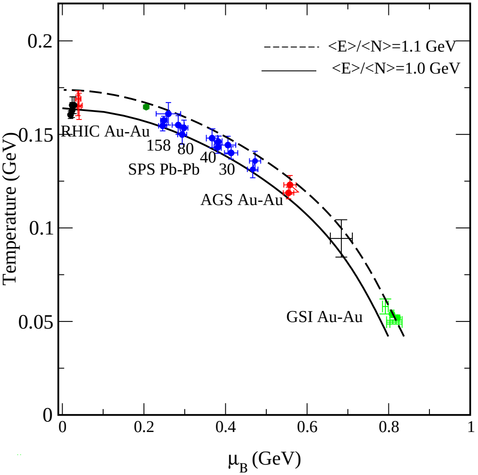
<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
RHIC Au-Au
SPS Pb-Pb
AGS Au-Au
GSI Au-Au
Temperature (GeV)
μB (GeV)
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1

E/N in 2005



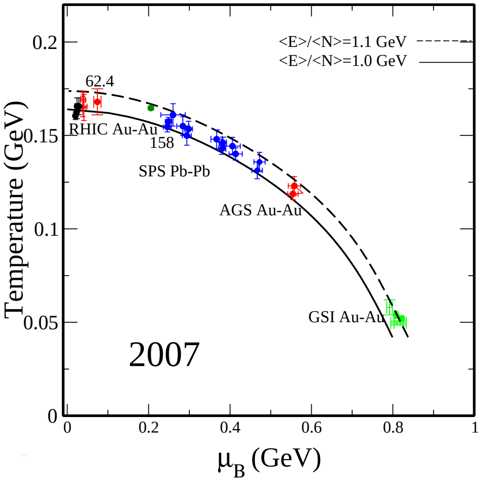
<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
RHIC Au-Au
158
80
40
30
SPS Pb-Pb
AGS Au-Au
GSI Au-Au
Temperature (GeV)
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
μB (GeV)

E/N in 2006



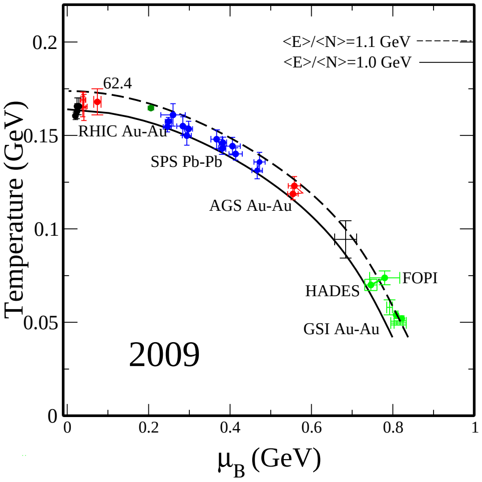
A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772, 167, 2006
J. Manninen, F. Becattini, M, Gazdzicki, Phys. Rev. C73 044905, 2006
R. Picha, U of Davis, Ph.D. thesis 2002

E/N in 2007

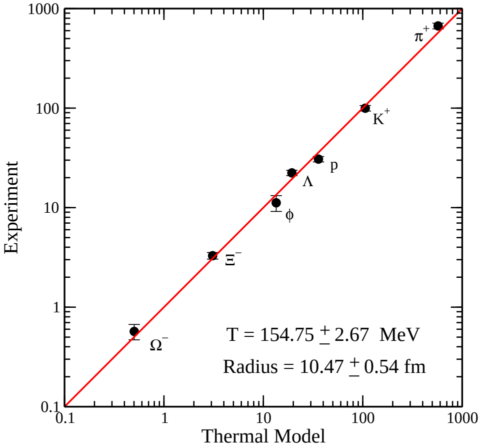


<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
62.4
RHIC Au-Au
158
SPS Pb-Pb
AGS Au-Au
GSI Au-Au
2007
Temperature (GeV)
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
μB (GeV)

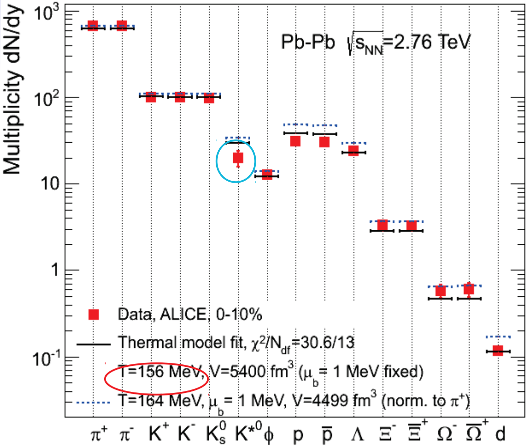
E/N in 2009

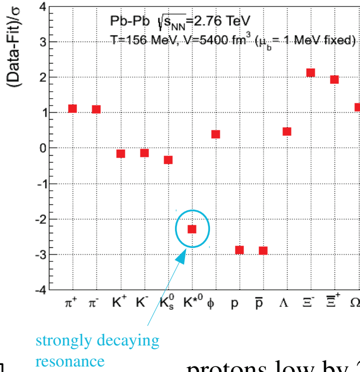

<E>/<N>=1.1 GeV
<E>/<N>=1.0 GeV
62.4
RHIC Au-Au
SPS Pb-Pb
AGS Au-Au
FOPI
HADES
GSI Au-Au
2009
Temperature (GeV)
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
μB (GeV)

2013



1000
100
10
1
0.1
0.1
1
10
100
1000
Experiment
Thermal Model
π+
K+
p
Λ
ϕ
Ξ−
Ω−
T = 154.75 ± 2.67 MeV
Radius = 10.47 ± 0.54 fm

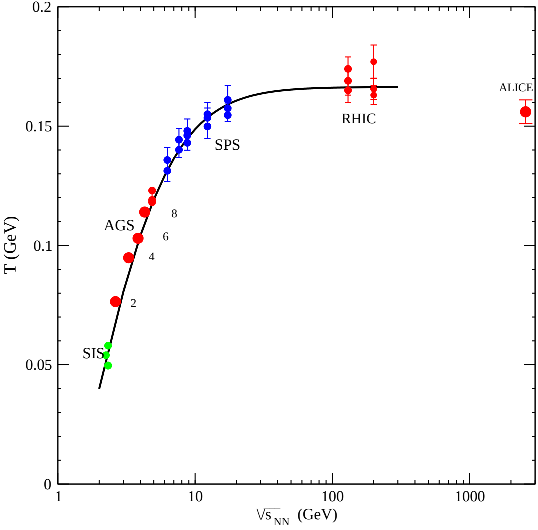




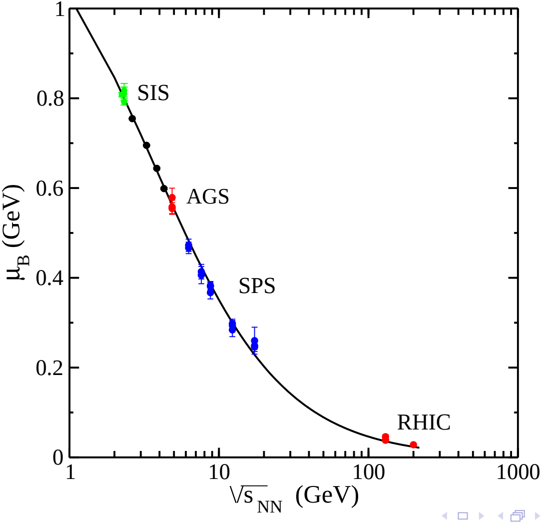
as compared to 2012: more and final data
T went from 152 to 156 MeV
red. χ^2 went from 4 to 2.35

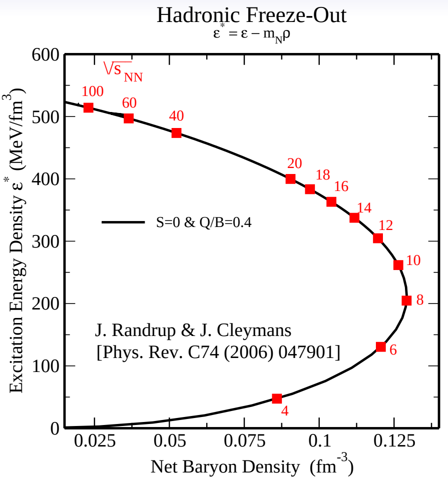
protons low by 2

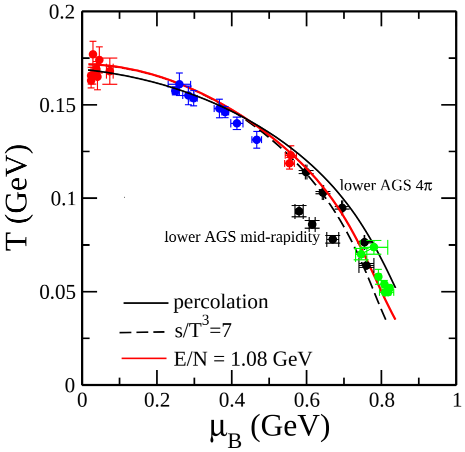
Chemical Freeze-Out Temperature

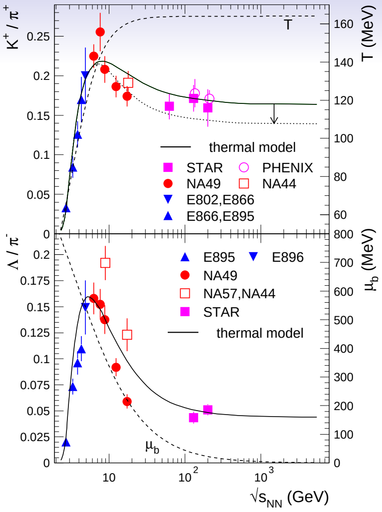


Chemical Freeze-Out μ_B




Hadronic Freeze-Out
ε* = ε − m_N ρ
√s_NN
100
60
40
20
18
16
14
12
10
8
6
4
S=0 & Q/B=0.4
J. Randrup & J. Cleymans
[Phys. Rev. C74 (2006) 047901]
Excitation Energy Density ε* (MeV/fm^3)
Net Baryon Density (fm^-3)
0
100
200
300
400
500
600
0.025
0.05
0.075
0.1
0.125


0.2
0.15
0.1
0.05
0
T (GeV)
0
0.2
0.4
0.6
0.8
1
μ_B (GeV)
lower AGS 4π
lower AGS mid-rapidity
percolation
s/T^3=7
E/N = 1.08 GeV



A. Andronic, P. Braun-Munzinger, J. Stachel, Physics Letters B673 (2009) 142.

In the thermal model the roller-coaster seen in the particle ratios corresponds to a transition from a baryon-dominated to a meson-dominated hadronic gas. This transition occurs at a

- temperature $T = 151$ MeV,
- baryon chemical potential $\mu_B = 327$ MeV,
- energy $\sqrt{s_{NN}} = 11$ GeV.

In the thermal model this transition leads to peaks in the $\Lambda/\langle\pi\rangle$, K^+/π^+, Ξ^-/π^+ and Ω^-/π^+ ratios but all occur at slightly different energies.
The maximimum in the K^+/π^+ ratio can be reproduced in thermal models but not the sharpness of the peak.
NEED SMALL STEPS IN BEAM ENERGY!!

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NEED SMALL STEPS IN BEAM ENERGY!!

The Tsallis Distribution at the LHC

Transverse Momentum Distribution

STAR collaboration, B.I. Abelev at al.
arXiv: nucl-ex/0607033; Phys. Rev. **C75**, 064901 (2007)
PHENIX collaboration, A. Adare et al.
arXiv: 1102.0753 [nucl-ex]; Phys. Rev. **C83**, 064903 (2011)
ALICE collaboration, K. Aamodt et al.
arXiv: 1101.4110 [hep-ex]; Eur. Phys. J. **C71**, 1655 (2011)
CMS collaboration, V. Khachatryan et al.
arXiv: 1102.4282 [hep-ex]; JHEP **05**, 064 (2011)
ATLAS collaboration, G. Aad et al.
arXiv: 1012.5104 [hep-ex]; New J. Phys. **13** (2011) 053033.

Transverse Momentum Distribution

STAR, PHENIX, ALICE, CMS, ATLAS use:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{T}\mathrm{d}y} = p_\mathrm{T} \times \frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nT(nT+m_0(n-2))}\left(1+\frac{m_\mathrm{T}-m_0}{nT}\right)^{-n}$$

What is the connection with the Tsallis distribution?
Also, the physical significance of the parameters n and T has never been discussed by STAR, PHENIX, ALICE, ATLAS, CMS.

Tsallis Distribution

Possible generalization of Boltzmann-Gibbs statistics

Constantino Tsallis
Rio de Janeiro, TBPF
J. Stat. Phys. 52 (1988) 479-487

Citations: 1 389
Citations in HEP: 513

ISSN 0029-3865

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CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Notas de Física

CBPF-NF-062/87

POSSIBLE GENERALIZATION OF BOLTZMANN-GIBBS STATISTICS

by

Constantino TSALLIS

RIO DE JANEIRO
1987

Entropy: Tsallis vs Boltzmann

The Boltzmann entropy is given by

$$S^B = -g \sum_i \left[f_i \ln f_i - f_i \right], \tag{1}$$

The Tsallis entropy is given by

$$S_T^B = -g \sum_i \left[f_i^q \ln_q f_i - f_i \right], \tag{2}$$

which uses

$$\ln_q(x) \equiv \frac{x^{1-q} - 1}{1-q}, \tag{3}$$

often referred to as q-logarithm.
By maximizing the entropy one obtains expressions for particle density, energy density and pressure.

Maxmizing the Entropy

$$f_j = \left[1 + (q-1)\frac{E-\mu)}{T}\right]^{-\frac{1}{q-1}} \tag{4}$$

For high energy physics a consistent form of Tsallis thermodynamics is:

$$S = -gV \int \frac{d^3p}{(2\pi)^3} \left[f^q \ln_q f - f \right], \quad (5)$$

$$N = gV \int \frac{d^3p}{(2\pi)^3} f^q, \quad (6)$$

$$\epsilon = g \int \frac{d^3p}{(2\pi)^3} E\, f^q, \quad (7)$$

$$P = g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E}\, f^q. \quad (8)$$

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor.

Thermodynamic consistency

$$dE = -pdV + TdS + \mu dN$$

Inserting $E = \epsilon V$, $S = sV$ and $N = nV$ leads to

$$d\epsilon = Tds + \mu dn$$

$$dP = nd\mu + sdT$$

In particular

$$n = \left.\frac{\partial P}{\partial \mu}\right|_T, \quad s = \left.\frac{\partial P}{\partial T}\right|_\mu, \quad T = \left.\frac{\partial \epsilon}{\partial s}\right|_n, \quad \mu = \left.\frac{\partial \epsilon}{\partial n}\right|_s.$$

are satisfied.

Transverse Momentum Distributions

In the Tsallis distribution the total number of particles is given by:

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} .$$

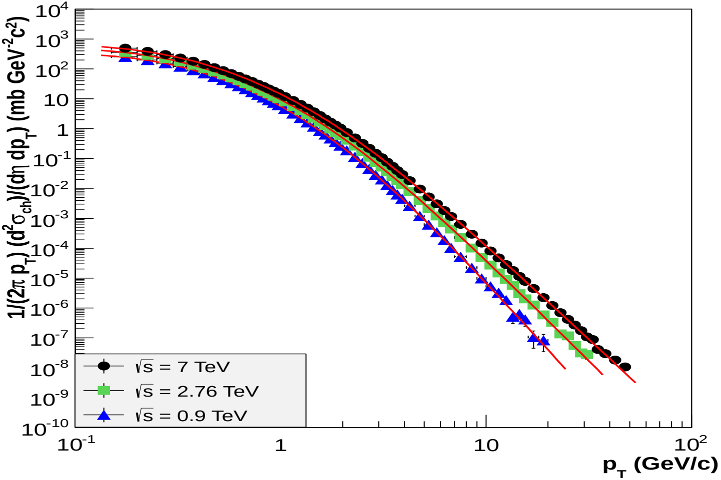
The corresponding momentum distribution is given by

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} ,$$

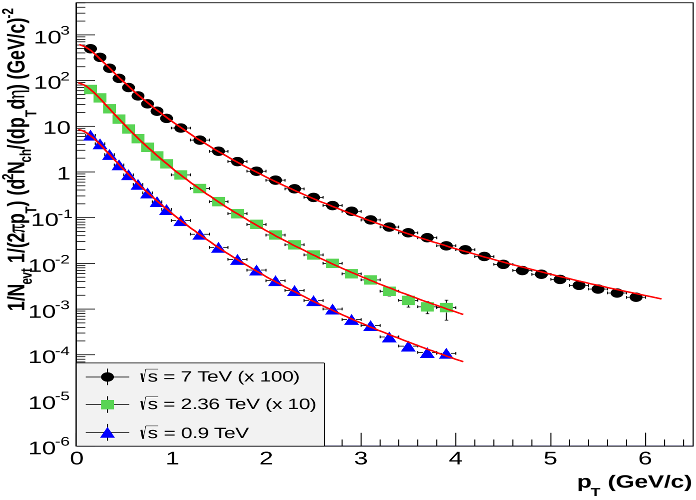
which, in terms of the rapidity and transverse mass variables, $E = m_T \cosh y$, becomes (at mid-rapidity for $\mu = 0$)

$$\left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}} ,$$

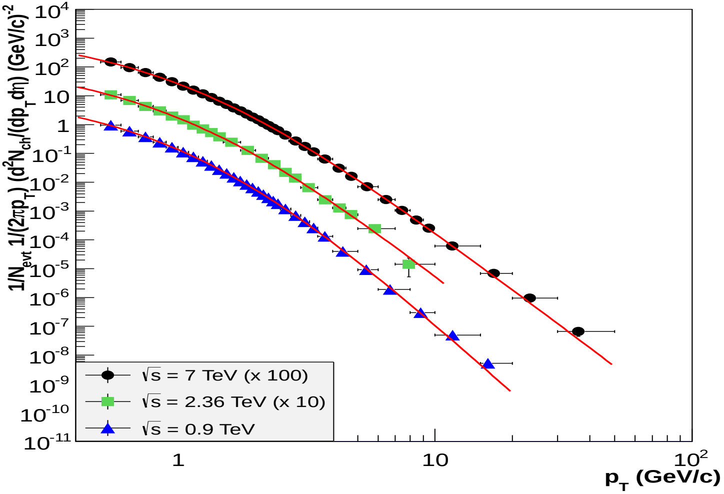
J.C. and D. Worku, J. Phys. **G39** (2012) 025006; arXiv:1203.4343[hep-ph].



ALICE, Phys. Lett. B **693**, 53 (2010); Eur. Phys. J. C **73**, 2662 (2013).

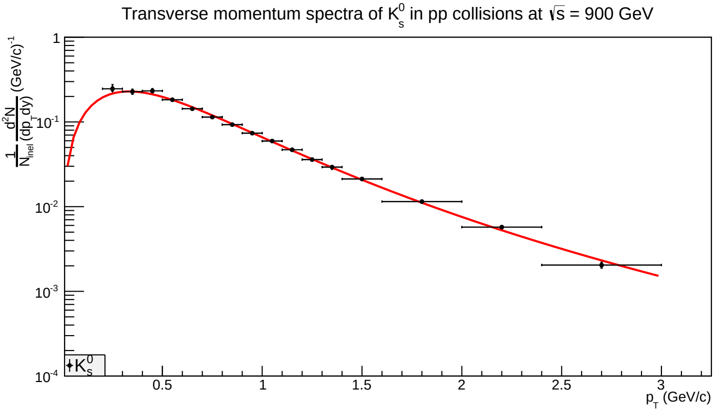


CMS, JHEP **02**, 041 (2010); Phys. Rev. Lett. **105**, 022002 (2010).



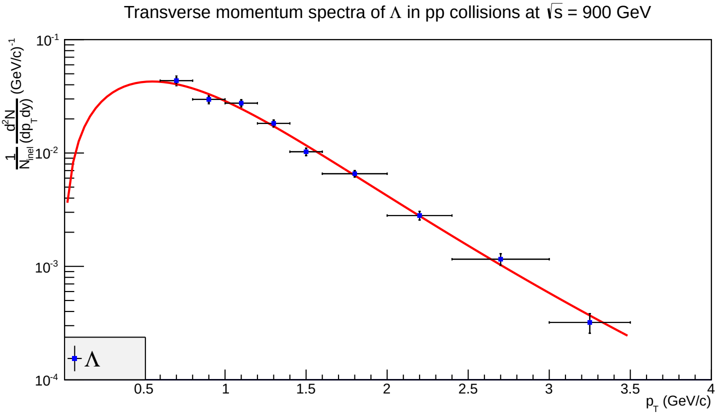
ATLAS, New J. Phys. **13**, 053033 (2011).

Tsallis Distribution p-p



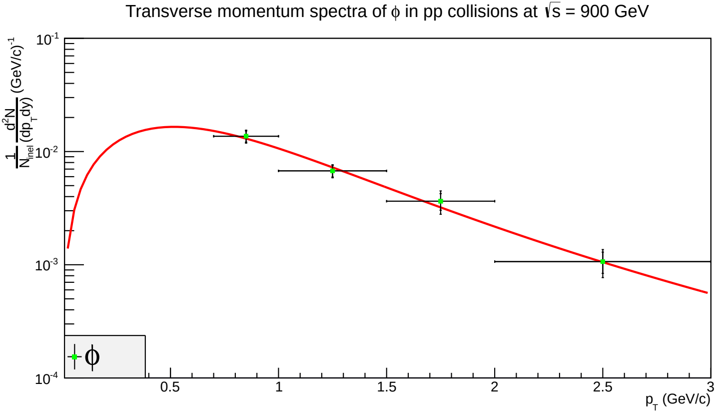
M.D. Azmi

Tsallis Distribution p-p



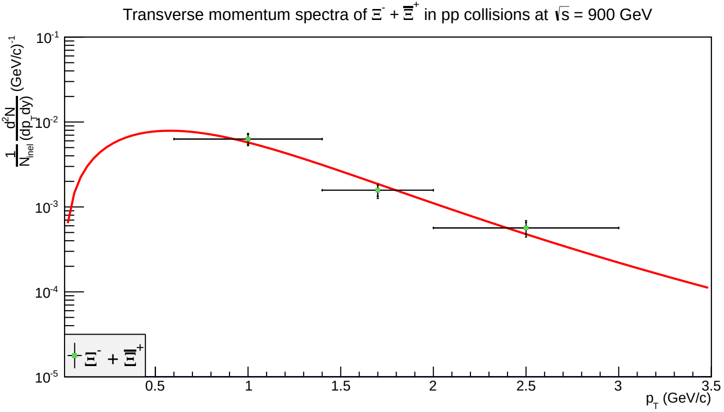
M.D. Azmi

Tsallis Distribution p-p



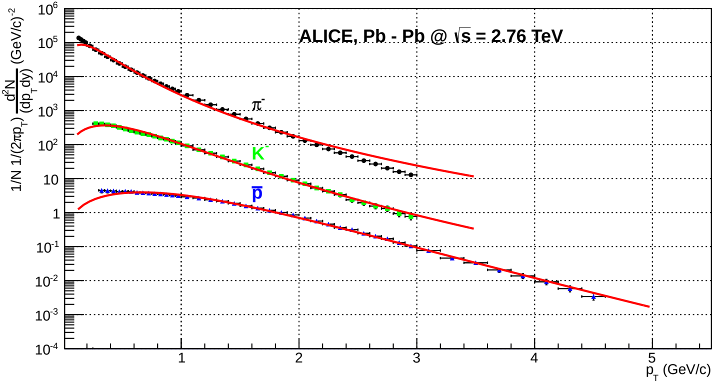
M.D. Azmi

Tsallis Distribution p-p



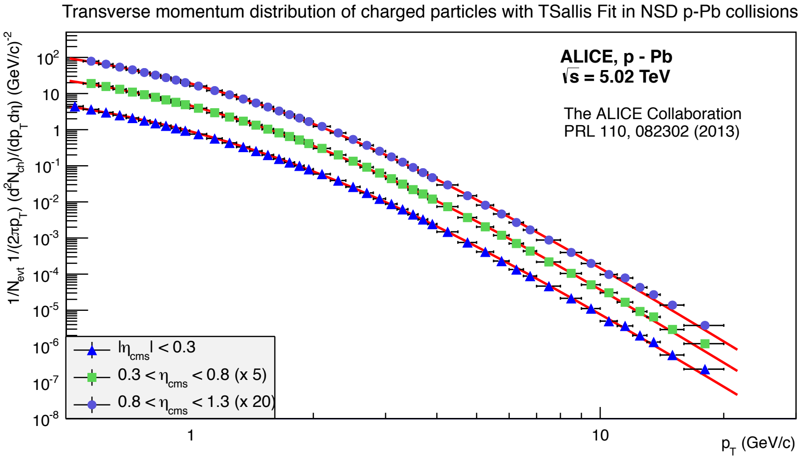
M.D. Azmi

Tsallis Distribution does not describe Pb-Pb

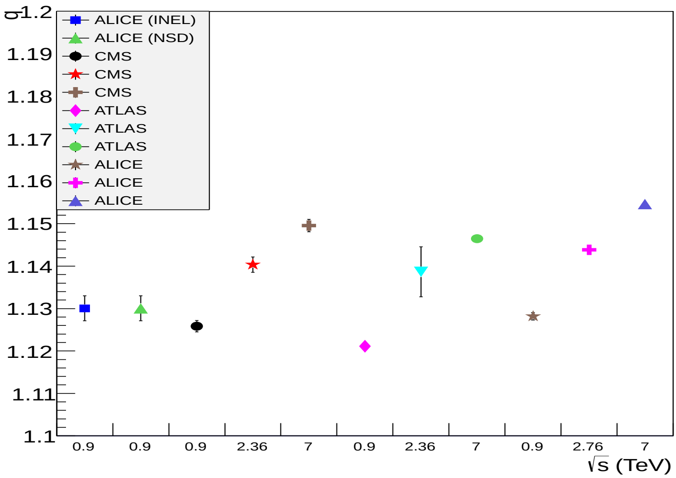


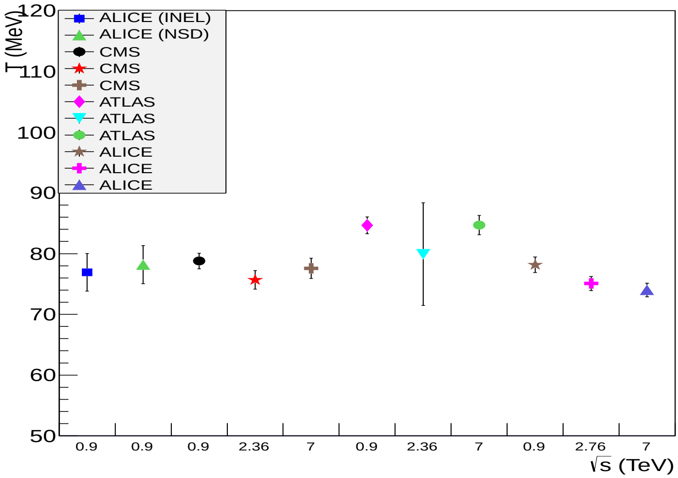
M. Danish Azmi

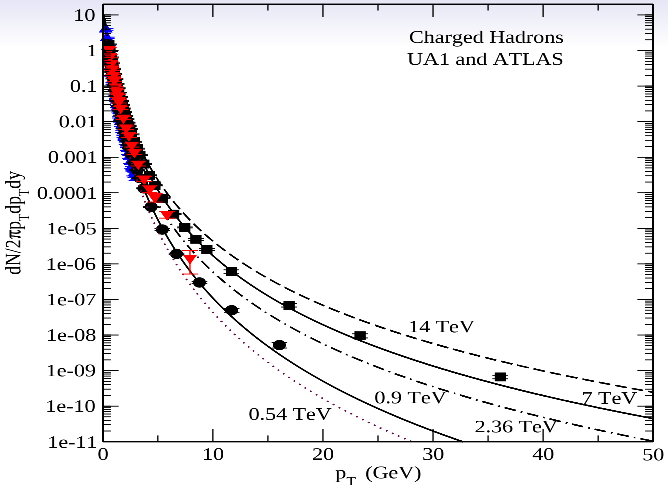
... but works for p-Pb collisions



$q = 1.140$ M. Danish Azmi


q
1.2
1.19
1.18
1.17
1.16
1.15
1.14
1.13
1.12
1.11
1.1
ALICE (INEL)
ALICE (NSD)
CMS
CMS
CMS
ATLAS
ATLAS
ATLAS
ALICE
ALICE
ALICE
0.9 0.9 0.9 2.36 7 0.9 2.36 7 0.9 2.76 7
√s (TeV)


T (MeV)
120
110
100
90
80
70
60
50
ALICE (INEL)
ALICE (NSD)
CMS
CMS
CMS
ATLAS
ATLAS
ATLAS
ALICE
ALICE
ALICE
0.9
0.9
0.9
2.36
7
0.9
2.36
7
0.9
2.76
7
√s (TeV)



J.C., G.I. Lykasov, A.S. Sorin, O.V. Teryaev, A.S. Pravan, D. Worku
Physics Letters B 723 (2013) 351-354.

Conclusion:

- Thermal models give a good description of particle yields from 1 to 7000 GeV.
- LHC: Temperature is a bit lower than expected.
- LHC: Too many pions and not enough protons.
- NICA: needed with small steps in beam energy! $\sqrt{s_{NN}} = 11$ GeV is special
- LHC: Final state in Proton-Proton collisions at the LHC produce a system consistent with Tsallis thermodynamics.